Critically Conscious Mathematics

We teach children from communities whose voices have historically been undervalued or ignored. We also believe that all human beings are valuable and deserve to be heard, particularly in a democratic society, and as such, our children need an education that activates their voices and ensures that they are able to advocate for themselves and their communities to be treated equitably, even or especially if that means reorganizing societal institutions—not just an education that helps them pass state tests. Ladson-Billings (1995) argues that instruction committed to collective empowerment must enable students to develop a) academic competence, b) cultural competence, and c) critical consciousness, and Gutiérrez (2009) writes that a transformative education is one in which students’ identity and agency are both developed (in a mathematical context, see: Martin, 2000; Cobb, Gresalfi, and Hodge, 2009).

Culturally responsive teaching is particularly important in secondary mathematics for three reasons:

- **Access and Agency** The language of power in our society is analytical and numerical; leading economic, financial, and political institutions are driven by quantitative models and forecasts, and academic research and consumer marketing alike often hinge their claims on statistics derived from mathematical analyses. Being able to interpret these claims and to model situations mathematically will give our children tools for identifying, advocating, and creating change (see Tate, 1995; Gutstein, 2006); “critical mathematical agency” exists when students “draw upon and construct mathematical understanding to investigate and critique situations in their lives and in the world around them, and... act transformatively upon those conditions” (Turner, 2003).

- **Opportunity** Instrumentally, curriculum and instructional methods that are relevant and responsive to students’ lived realities are likely to increase their engagement in mathematics that they may otherwise perceive to be dry, abstract, or boring when it is taught traditionally. This gives them a greater chance of developing deep and enduring interest and understanding in a field that often leads to educational and career opportunities that are likely to be meaningful, stable, lucrative, and/or prestigious.

- **Mathematics** It makes children better mathematicians: Schoenfeld (1992) and others (see Brenner, 2011; McIntosh, 2011, etc.) posit that true mathematicians not only solve problems given to them by applying algorithms, but also find problems to solve. They are curious, they question the world around them, and they have the quantitative and analytical tools to determine whether what they observe is “normal” and “fair”.

Critically conscious mathematics can be understood as the intersection of what is being taught—the curriculum—and how it’s being taught—the pedagogy. Content must be mathematically rigorous, both in terms of what is taught and how it is taught, and:

- **Curriculum** Teachers must situate their content in context (through word problems, performance tasks, projects, lesson hooks, and instructional investigations, etc.) that is:
  - Relevant to students’ interests and lives (e.g. using the subway to teach positive and negative integers in districts with strong public transit; comparing proportions of free throw shots made by a hometown hero vs. star on rival team, etc.)
  - Responsive to their interests and to current events in the community around them (e.g. using surveys to forecast election results; triangulating the best place to locate a new grocery store in a community; calculating payday lender rates, etc.)
  - Broadens their understanding of social issues (e.g. sweatshop wages, income tax distribution, housing discrimination, etc.)
- **Pedagogy** Instructional strategies must be selected for their capacity to validate student knowledge [e.g. learning builds on what students know], encourage student voice [e.g. real & meaningful discourse], and develop student initiative [e.g. students exhibit mathematical agency by questioning and challenging the teacher and/or peers, directing their own learning, etc.]. Learning in mathematics may be integrated with other subject areas [e.g. scientific field lesson collecting soil samples + mathematical analysis of toxin concentration + history of environmental activism + ELA persuasive paper].

**Sources and Additional Reading**


### Handout 2: Culturally Responsive Curriculum – Some examples

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<thead>
<tr>
<th>Method*</th>
<th>Example</th>
<th>How does this choice contribute to:</th>
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<tbody>
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<td><strong>Academic achievement?</strong></td>
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<td><strong>Using familiar modes of communication</strong> [call and response, vocabulary in a home language, rewriting lyrics to popular songs]</td>
<td>A teacher in a classroom with many ELL students encourages them to take notes in their home language, and to share the names of various polygons in their home language with the whole class. Students write rap songs to describe a mathematical process or idea.</td>
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<td>Making <strong>analogies</strong> between new content and students’ lived realities</td>
<td>In Boston, Algebra Project teachers teach students about positive and negative integers by riding the subway together. On the train, they solve math problems and discuss whether the government should subsidize public transit, and about where in Boston students live and spend their time (and where they don’t)</td>
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<td>Regularly bringing in <strong>community members</strong> to share math-related expertise [from their jobs or home lives]</td>
<td>A grandmother who is renowned in the community for her baked goods teaches elementary students a lesson on baking piecrust, and students use ratios and proportions to calculate the ingredients needed to make different pies for a community event. A firefighter explains how fire stations are strategically located such that no home in the community is more than 15 minutes away from a fire station, and so that there is a proportional number of fire stations for the total number of homes in the community.</td>
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(Continued on the next page) *Note that these methods can be combined with one another or with other themes and projects during a course; nothing is mutually exclusive, and this is by no means a comprehensive list. Instead, it is intended to serve as a starting point for brainstorming and reflection.*
<table>
<thead>
<tr>
<th>Method*</th>
<th>Example</th>
<th>How could this choice contribute to:</th>
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<tr>
<td>Choosing contexts for their relevance to students’ lived realities or for the opportunity to explore broader societal issues when framing <strong>word problems</strong></td>
<td>Ask “Factory workers aged 14, 15, and 16 in Honduras make McKids children’s clothing for Wal-Mart. Each worker earns 43 cents an hour and works a 14-hour shift each day. How much does each worker make in one day, excluding fees deducted by employers?” instead of “A group of youth aged 14, 15, and 16 go to the store. Candy bars are on sale for 43¢ each. They buy a total of 12 candy bars. How much do they spend, not including tax?” (Gutstein &amp; Peterson)</td>
<td>Academic achievement?</td>
</tr>
<tr>
<td>Choosing contexts for their relevance to students’ lived realities or for the opportunity to explore broader societal issues when framing <strong>instructional tasks or projects</strong></td>
<td>When comparing functions, use data on population growth to determine which type of function best fits the data; use it to make predictions about the solvency of Social Security. When learning about data and statistics, students design a survey about a local issue and conduct the survey in their neighborhood; analyze results.</td>
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<td>Choosing an essential question or enduring understanding that serves as a <strong>unifying theme</strong> for explorations, mini-lessons, and/or tasks over multiple lessons or the unit</td>
<td>A teacher chooses the EQ of “what’s fair?” Lessons throughout the course explore what mathematical models would predict about obesity rates, educational attainment, median income, subprime lending rates, and incarceration rates, and compare these to real statistics about particular populations to determine whether any discrepancies are random or could provide evidence of injustice or institutional discrimination. See Handout 2b for an example of how a teacher dives deeply into themes in yearlong courses.</td>
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Credit Cards: Friend or Foe?

Take a look at the Credit Card offer your group received. Read it carefully and identify the important things to know regarding interest rates, annual fees and "fine print."

- 8.90% annual percentage rate for first 6 months then 15.24%
- If you don't pay, annual percentage rate is 21.24%
- Late Fees: $19 on balances ≤ $400 and $38 on balances > $400

You purchase a computer for college for $1,000 using this credit card. Read each situation below carefully. Solve each for your credit card offer.

1. Oops! You got your first bill but it got lost in the pile of papers on your desk so you didn’t pay. How much is your bill the next month? Show all work.

   \[
   1,000 \left(1 + \frac{0.21}{12}\right) \approx 1,017.86 \approx 1017.86 + 38 = \$1055.86
   \]

2. You forgot again! How much is your next month’s bill? Show all work.

   \[
   1055.86 \left(1 + \frac{0.21}{12}\right) \approx 1074.72 \approx 1074.72 + 38 = \$1112.72
   \]

3. Okay, new scenario. The first bill came for $1,000. You remembered to pay it. You paid the minimum of $25. How much is the next month’s bill. Show all work.

   \[
   1,000 - 25 = 975 \approx 975 \left(1 + \frac{0.09}{12}\right) \approx 982.26
   \]

   How much of the original cost of the computer did you pay? 17.74
   How much of what you paid went to interest? 7.26
4. How long will it take you to pay off the whole amount? Show your work.

\[
p = \frac{757.26 e^{(0.0896 \cdot \frac{1}{12})}}{1 - e^{-(0.0896 \cdot \frac{1}{12})}} \approx 964.39\]

56 months
4 years + 8 months

Lost payment:

\[
56 \times 25 = 16.05
\]

5. How much will you have paid in total? Show your work.

\[
56 \times 25 = 1,400\]

\[
+ 16.05
\]

6. How much of that was interest? Show your work.

\[
1,400 - 1,000 = 400\] in interest

Write up your findings: Your write-up should include:

What did you learn about credit cards?
Would you recommend this particular credit card? Why or why not?
How has your opinion about credit cards changed from this investigation?
Will you apply for a credit card? If so, what kind of card are you looking for? If not, why not?
How did this investigation help your understanding of the mathematics of exponential functions and interest rates?
I learned that having a credit card can be a hassle. When you buy something with a credit card, you may actually be paying more because some of what you paid goes to interest. It actually all depends on the credit card because different ones offer different things. It’s annoying if you are late on the payment too. You have to pay more money. If you forget to pay, your annual percentage rate will increase (a lot with this particular credit card I have). It will jump to the default default APR. And you can’t forget the late fees either.

I would not recommend this particular credit card because you will lose a lot of money on the interest. If you paid the first bill for $1,000 and pay the minimum of $25, you only pay $17.74 and $7.26 goes to interest. Also, the default APR is very high and you have to pay $19 if your balance is less than $400 and $38 if your balance is greater than $400 for late fees. I think there are better credit card offers.

I always wanted a credit card because I thought you could just buy anything you wanted. They always show happy people buying a lot of stuff with their credit card with no worries or Now I know it’s not that easy. A credit card is a way to get more loans which you eventually have to pay back. I don’t think I would want one. It’s annoying. I would rather apply for a debit card with they take money straight from your account. A smart choice.

This has helped me understand credit cards a little better. A credit card is an exponential function and has a limit. The APR interest rate gets smaller, explodes.
Handout 2b: Framing an entire course around a particular lens

Advanced Algebra’s Journey:

Mathematics: Recursive Formulas & Shifted Geometric Sequences
Social Issue: Is racism involved in lending practices in Minneapolis?
Student groups had to write a recursive formula to model a mortgage loan. We all bought the same house (using the current median home value in our community). Every group was given a different interest rate and groups had to generate a monthly payment that would pay their loan off in exactly thirty years. Groups reported back their monthly payment, total amount paid for their home, and the minimum income they would need to make to avoid becoming a “cost-burdened home” (meaning the mortgage payment can be no more than 30% of their income). After presenting all the groups findings, we looked at a recent report about subprime lenders in Minneapolis. This led to our next investigation.

Mathematics: Standard Deviation
Social Issue: Is Minneapolis Segregated?
Student groups were assigned a neighborhood, which was also broken down into smaller communities. Students had to determine if their neighborhood was segregated based on the number of standard deviations each community was away from the city’s percentage for designated racial categories. Student groups had to make a decision about their neighborhood based on the mathematics. Each group created a large poster including a neighborhood map, a standard deviation table, and segregation statement and we did a gallery walk. Then as a large class we had a discussion about Minneapolis. I was very careful not to tell students “yes” or “no.” It was critical for the students themselves to understand the mathematics to make their decision. This was a particularly powerful challenge for the students because many could easily calculate standard deviation but many struggled to find a realistic application for it.

Mathematics: Linear Models (scatterplots, lines of best fit, writing equations, making predictions)
Social Issue: Do the demographics of a neighborhood have implications for its residents?
Students were given data about graduation rates and median income levels based on the “neighborhood school district zones.” Students created a scatterplot, draw a line of best fit, and had to make a series of predictions. This was a great challenge because it provided many teachable moments including scale choice, correlation vs. causation, point-slope form, etc. It was also an amazing challenge socially for the students. Many students started to understand and digest how racism exists as an institution vs. individual choice. Students also became more verbal about seeing themselves in the curriculum in their reflections.

Mathematics: Exponential and Logarithmic Functions
Social Issue: Is College worth it?
Each student group was assigned a different college or university in Minnesota and had to model a student loan based on the average debt of a graduate. Interest rates were based on current offers for private loans from US Bank and Wells Fargo. We were able to create a student loan debt guide for our college and career room. We also looked at affordable payments. We created neighborhood posters that displayed an affordable payment for each community and color-coded the total amount paid for a loan. This activity was based on MN average student loan debt and the interest rates used in the initial activity.

Mathematics: Linear Programming
Social Issue: How do we best manage an awareness campaign?
At the time of the challenge, Integration Funding for the public schools was being reviewed. This is a major source of funding for the Minneapolis Public Schools. Students were given time and financial constraints. They had to write inequalities to represent these constraints and then find the optimal number of people they could get to come out to a rally based on other given information.

Mathematics: Expected Value
Social Issue: Is it okay to use a regressive tax to fund a new Viking’s Stadium?
We analyzed the most recent plan to fund the Viking’s football stadium. The plan was to generate revenue through a 3% tax on pull-tabs. Students had to calculate the expected value for different pull-tab games (currently being sold in MN). They used this to generate a conservative number of pull-tab games that would need to be sold in order to produce the numbers promised by plan proponents. We also looked at reports about pull-tab player demographics and looked at the location of new proposed sites.
Intermediate Algebra's Journey

Mathematics: Linear Regression
Social Issue: Is Minneapolis Segregated?
Students drew a line on a map of the city. They then calculated the demographics of each neighborhood they travelled through. Students compiled results and created a class scatter-plot that graphed distance vs. percent of population. Students then found the line of best fit and compared the lines to the constant functions that would result if all neighborhoods were equally representative of the city’s demographics. Upon completion of the challenge students were shocked by the differences but many students maintained that the city was not ‘segregated’ since that would require a law stating when certain race had to live. I pushed the students to explain the differences in demographics across the city and their almost unanimous response was that personal choice was the main factor. This limited perspective informed our next challenges as I began to involve students in a conversation about the impacts of neighborhood structures on the individual.

Mathematics: Linear Inequalities
Social Issue: Do poor neighborhoods have fewer high school graduates?
In preparing students for a discussion about why segregated neighborhoods matter, I found an online article that detailed a study maintaining that households with lower median incomes produced fewer high school graduates. I challenged the students to explore whether or not this was true in our own community. Students calculated the median incomes of neighborhoods surrounding our big 6 high schools and found a high correlation with graduation rates. I asked students to explain why students from poorer neighborhoods graduated at a rate below others and their response was again surprising. They again explained this disparity as a personal choice often impacted by poor parenting. I realized my students did not have a perspective on how societal constructs impacted the individual or how personal choices can affect society at large.

Mathematics: Exponential Functions
Social Issue: Projected future income based on educational attainment
Students used a graphic on the U.S. Department of Education website to graph a function that related educational attainment to future average income. Students were also challenged to explain how high school dropouts impacted the U.S. economy through several statistics. This helped students to begin to develop a perspective beyond one of individual choice. We also read an article about how different programs were helping high school dropouts break the cycle of poverty. Much to my surprise, many of my students had never heard this term before.

Mathematics: Function Notation
Social Issue: Who does the cycle of poverty affect?
Students were asked to graph graduation rates of our own district by race over the course of the past several years. They were also asked to make predictions as to when, if ever, all graduation rates would be equal. In the reflection I asked students to combine the social concepts of all previous challenges with this one. I found the responses this time to be much more socially aware. Students asked, “If graduation rates are linked to increased chance of poverty, why aren’t we doing anything about it?”

Mathematics: Quadratics
Social Issue: Incarceration Rates
Over the MLK Jr. break I asked students to read an editorial about how Dr. Martin Luther King, Jr.’s Dream had yet to be realized in Minnesota. They were particularly struck by one line in the editorial that claimed the nation had more people of color in prison than in college. Our next challenge naturally lent itself to discovering whether or not this was true. Students graphed incarcerations rates and created quadratic models to describe the changes over time. They, in turn, used these models to make predictions.

Mathematics: Probability
Social Issue: Judicial fairness and the Jena 6
Students naturally wondered why incarceration rates were so disparate. They wondered whether issues of police-public contacts and racism in the judicial system were issues at play. I gave them access to data available in Minnesota that highlighted such disparities. We also explored the case of the Jena 6 and the probability of one of the defendants receiving an all white jury.